Stopper Effects in Network Type Polymers

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The specific parameters describing the flows of Bingham type rheological fluids through a circular pipe, under the action of a pressure gradient in the direction of the movement are established, using the Non-Standard Scale Relativity Theory approach. In such context, an analytical solution and a numerical application (for KELTAN 4200 — an etilene-co-propilene copolymer) are obtained. The friction effort has two components, one of specific fluid gliding, the other of shearing, depending on transverse speed gradient. In the central area of the fluid movements a particles agglomeration (fluid stopper) occurs, defined by a constantly moving structure.

Keywords: rheology, friction effort, polymers, fluid stopper, speed

A great variety of materials is categorized as complex fluids: polymers (elastomers, thermoplastics, and composites) [1], colloidal fluids, biological fluids (DNA - who creates cells by means of a simple but very elegant language and it is responsible for the remarkable way in which individual cells organize into complex systems like organs and these organs form even more complex systems like organisms -, proteins, cells, dispersions of biopolymers and cells, human blood), foams, suspensions, emulsions, gels, micelar and liquid-crystal phases, molten materials, etc. Therefore, fluids with non-linear viscous behaviours, as well as viscoelastic materials are complex [2-3].

Particle dynamics in complex fluids is highly nonlinear. For example, the formation of amorphous solids (glasses, granular or colloids) do not comply with the physical mechanism explaining solids crystallization. So, in amorphous solids, either lowering the temperature or increasing the density, the dynamic process achieves a level where the system cannot totally relax and therefore becomes rigid. This phenomenon is known as *glass transition* (when the temperature lowers) or *jamming transition* (when density increases) [4,5]. Also, the stress of a viscoelastic fluid, unlike the Newtonian fluid, depends not only on the actually stress applied, but on the one applied during previous deformation of the fluid [6].

In order to develop new theoretical models we must admit that the complex fluids systems that display chaotic behaviour are recognized to acquire self-similarity (space-time structures seem to appear) in association with strong fluctuations at all possible space-time scales [1-3]. Then, for temporal scales that are large with respect to the inverse of the highest Lyapunov exponent, the deterministic trajectories are replaced by a collection of potential trajectories and the concept of definite positions by that of probability density [7,8].

Since the non-differentiability appears as a universal property of the complex fluids systems, it is necessary to construct a non-differentiable physics. In such conjecture, by considering that the complexity of the interactions processes is replaced by non-differentiability, it is no longer necessary to use the whole classical "arsenal" of quantities from the standard physics (differentiable physics).

This topic was developed in the Scale Relativity Theory (SRT) [7,8] and in the non-standard Scale Relativity Theory (NSRT) [9-23]. In the framework of SRT or NSRT we assume that the movements of complex fluids entities take place on continuous but non-differentiable curves (fractal curves) so that all physical phenomena involved in the dynamics depend not only on the space-time coordinates but also on the space-time scales resolution. From such a perspective, the physical quantities that describe the dynamics of complex fluids may be considered fractal functions [7,8,]. Unlike the classical case previously studied [24], the entities of the complex fluids may be reduced to and identified with their own trajectories, so that the complex fluids will behave as a special interaction-less "fluid" by means of its geodesics in a non-differentiable (fractal) space (Schrödinger or hydrodynamic forms).

In the present paper, we propose the NSRT approach to analyze the complex fluids dynamics. Particularly, we determine the parameters that characterize the Bingham type fluid flows, through a horizontal pipe with circular section. The study contains an analytical solution and a numerical application, using the Navier-Stokes type equations, from the NSRT approach, in cylindrical coordinates and the friction effort for a Bingham type fluid. Our numerical mathematical model differs than other models used to describe the Bingham fluids [25,26].

Experimental part

The polidispersed heterogeneous mixtures that have fluid continuous phase can also have a discontinuous phase given by solid or fluid particles with different properties, such as density, granulometry and shape. The

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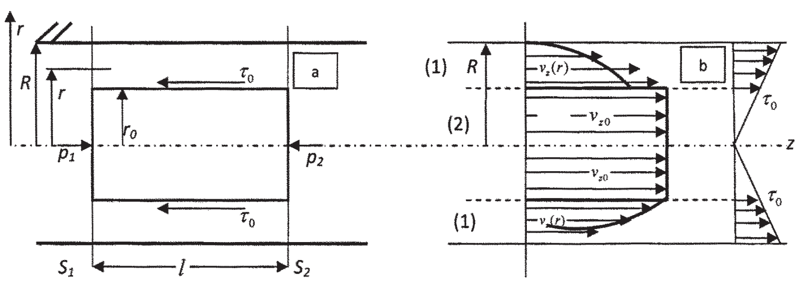

Fig. 1a,b. Pressure gradient flow of a rheological fluid through a circular pipe(a). Speed and effort diagrams(b)

discontinuous phase of these heterogeneous mixtures, in different working conditions (depositing, transport, phase separation), do not have unitary behaviour that could exactly be characterized [2-3]. In the majority of cases, the discontinuous phases in a heterogeneous mixture with laminar flow through a circular pipe or linear flow with flow direction change (bends, speed limiters, reductions) concentrate towards the axis of the pipe with a distribution that is proportional to the size particle[21,22].Therefore, these fluids are complex molecular structures that do not obey Newton's law, which are called rheological fluids (complex fluids)[2-3]. The Bingham fluids are included in this category[25,26].

The non-Newtonian fluids are found in many technical applications in petroleum engineering, civil engineering, environmental engineering, food engineering as muds, paints, cement pastes, slurries, food substances, etc.[25-28].

Behaviours of the Bingham type complex fluids

When Bingham type fluid moves, the energetic and kinematic characteristics of the flow differ than those of the Newtonian fluid. Thus, due to viscosity tangential unitary effort variation law [25,26],

$$\tau = \tau_0 + \eta \frac{dv}{dn} \qquad (1)$$

the distribution of speeds in transverse section of a pipe covers two sub-domains (figs. 1a,b).

In the central area of radius $r \in [0; r_0]$, the unitary effort is lower in value than the flow limit τ_0 . Therefore the Bingham type fluid moves as an apparently undistorted rigid system, having the shape of a stopper with quasi-parallel walls to those of the pipe.The solid stopper flows with constant speed in the central area of the pipe, without changing its structure. The radius r_0 the two sub-domains border, depend on the rheological characteristic of the Bingham type fluid.

In sub-domain $r \in [r_0; R]$, the effort τ exceeds the value τ_0 and the Bingham type fluid flows (a layer with finer particles and lower concentration appears).

From the energetic point of view, the motion of Bingham plastic type fluid implies additional external forces as the Newtonian fluids. Thus, supplementary energy consumption is achieved.

The rheological behaviours of suspensions entail the following aspects [25-26]:

- highlighting the factors related with system's structure (composition, viscosity of the two phases, dimensional distribution of the particles, the nature of the stabilizer, etc.);
- viscosity dependence on shear speed;
- viscosity dependence on sample history or, more specifically, on time effect as a consequence of structuring and de-structuring processes, known as rheopexy and thixotropy.

At lower concentrations[1], the rheological behaviours of a suspension are Newtonian if the dispersion medium is also a Newtonian one. The Newtonian behaviours imply the following characteristics:

-the only tension that is generated in the simple shear flow is shear tension τ, the differences between two normal tensions being null;

- shear viscosity is independent on shear speed;
- viscosity remains constant during the searing process, while tension tends to zero, as soon as shear stops;

-according to the relation between viscosity tangential effort τ and the speed gradient dv/dz, respectively to the shear speed dv/dt, fluids can be classified in Newtonian and non-Newtonian ones[27-29];

-Non-Newtonian fluids are those fluids that do not obey Newton's viscosity law, i.e. they do not any longer submit to proportionality between the tangential effort τ and the speed gradient dv/dz.

Similarly with the Newton law, the rheological behaviours of some real fluids are described by the relation[1-3, 30]:

$$\tau^n = k_a \frac{dv}{dz} \qquad (2)$$

where k_α is the apparent viscosity coefficient and n an integer number. For $n = 1$ and $k_a = \eta$ from (2) the Newton's law results.

Rheological fluids have certain characteristics such as:

- elasticity and delayed elasticity;
- plasticity and dynamic plasticity;
- viscosity, relaxation, post effect;
- thixotropy, rheopexy.

Results and discussions

Mathematical model

It is well known that in the NSRT approach [10, 13, 14], the dynamics of the complex fluids are described by the fractal operator:

$$\frac{\hat{d}}{dt} = \frac{\partial}{\partial t} + \hat{V} \cdot \nabla - iD(dt)^{\left(2/D_F\right)-1} \Delta \qquad (3)$$

where:

$$\hat{V} = V_D - iV_F \qquad (4)$$

is the complex velocity, V_D is the differentiable and independent scale resolution velocity, V_F is the non-differentiable and dependent scale resolution, $\hat{V} \cdot \nabla$ is the convective term, $D(dt)^{(2/D_F)-1}\Delta$ is the dissipative term, D is the "diffusion" coefficient associated to fractal-non-fractal transition, dt is the time scale resolution and D_F s the fractal dimension. For D_F any definition can be used (Kolmogorov fractal dimension, Hausdorff-Besikovici fractal dimension etc. [7,8]), but once accepted such a definition for D_F, it has to be constant over the entire analysis of the complex fluid dynamics. In this conditions the Navier-Stokes type equation becomes:

$$\frac{\partial \hat{V}}{\partial t} + \left(\hat{V} \cdot \nabla\right)\hat{V} = -\frac{\nabla p}{\rho} + \tilde{D}\Delta\hat{V} + f \qquad (5)$$

with

$$\tilde{D} = \nu - iD(dt)^{\left(2/D_F\right)-1},$$

or, even more, by separating the real part and the imaginary one (i.e. separation on resolution scales):

$$\frac{\partial V_D}{\partial t} + (V_D \cdot \nabla)V_D - (V_F \cdot \nabla)V_F - D(dt)^{\left(2/D_F\right)-1}\Delta V_F = = -\frac{\nabla p}{\rho} + \nu\Delta V_D + f$$

$$\frac{\partial V_F}{\partial t} + (V_F \cdot \nabla)V_D + (V_D \cdot \nabla)V_F + D(dt)^{\left(2/D_F\right)-1}\Delta V_D = \nu\Delta V_F \qquad (6\ a,b)$$

In the relations (3)-(5 a,b) p is the pressure, ρ is the density, ν is the kinematic viscosity, $\tilde{D}$ is the generalized kinematic viscosity and f is the specific force. If the motions at fractal scale resolution are irrotational, i.e.

$$V_F = D(dt)^{\left(2/D_F\right)-1}\nabla \ln\rho \qquad (7)$$

the equation (6b), by integration and choosing a null value for the integration constant, takes the form of a continuity type equations

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho V_D) = \eta\nabla \cdot (V_F) \qquad (8)$$

where $\eta = \rho\nu$ is the dynamic viscosity. Particularly, for ρ = const. the equations (6a) and (8) become:

$$\frac{\partial V_D}{\partial t} + (V_D \cdot \nabla)V_D = -\frac{\nabla p}{\rho} + \nu\Delta V_D + f$$

$$\nabla \cdot (V_D) = 0 \qquad (9\ a,b)$$

The momentum equations (6a), together with the continuity type equation (8) allow the solving of movement problems of real fluids if the limit conditions are known.

Let us consider the unidirectional flow, with speed $V_D \equiv v$ of a fluid through a cylindrical pipe, with radius R, under the action of a pressure gradient (figs. 1a,b). Thus the dynamics will be analyzed using equations (9 a,b).

Under these circumstances,

$$v_r = v_\varphi = 0;\ v_z \neq 0$$

If we neglect the specific forces,

$$f_r = f_\varphi = f_z = 0,$$

and using the continuity equation in the imposed conditions, the dynamic equilibrium equation results:

$$\eta\frac{d^2 v_z}{dr^2} + \frac{1}{r}\frac{dv_z}{dr} = \frac{dp}{dz} \qquad (10)$$

Considering the expression of the friction effort for Bingham type fluid(1), it results:

$$\frac{\partial\tau}{\partial r} + \frac{\tau}{r} = \frac{\partial p}{\partial z} \qquad (11)$$

The flow domain can be separated into two distinct sub-domains. In sub-domain (1) for $r \in [r_o;\ R]$, where the shear speed is high so the fluid tends to have Newtonian behaviours. In sub-domain (2) for $r \in [0;\ r_o]$, the fluid constantly moves as a solid stopper so,in this area, the deformation tension τ_o was not exceeded and the fluid is not sheared.

Let us calculate the relations of flowing speed in the two areas, under the action of the pressure gradient, through the boundary conditions, both for the flowing speed and the shear speed dv_z/dr.

The solution of equation (10) is:

$$v_z(r) = \frac{\Delta p}{4 \cdot \eta \cdot l}r^2 + \frac{\tau_0}{\eta}r + c_1 \ln r + c_2 \qquad (12)$$

where c_1 and c_2 are integration constants. The values of these constants are established by the following boundary conditions:

i) for $r = r_o$, i.e. on the stopper borderline, $dv_z/dr = 0$, so we will have:

$$\frac{dv_z}{dr} = \frac{2 \cdot r \cdot \Delta p}{4 \cdot \eta \cdot l} + \frac{\tau_0}{\eta} + \frac{c_1}{r} \qquad (13)$$

$$\frac{r_0^2 \cdot \Delta p}{2 \cdot \eta \cdot l} + \frac{\tau_0 \cdot r_0}{\eta} + c_1 = 0 \qquad (14)$$

$$c_1 = -\frac{\Delta p}{2 \cdot \eta \cdot l}r_0^2 - \frac{\tau_0}{\eta}r_0 \qquad (15)$$

$$v_z(r) = \frac{\Delta p}{4 \cdot \eta \cdot l}r^2 + \frac{\tau_0}{\eta}r - \frac{\Delta p}{2 \cdot \eta \cdot l}r_0^2 \ln r + \frac{\tau_0}{\eta}r_0 \ln r + c_2 \qquad (16)$$

ii) for $r = R$ i.e. at the wall, $v_z(R) = 0$.

$$\frac{\Delta p}{2 \cdot \eta \cdot l}\left[\frac{R^2}{2} - r_0^2 \ln R\right] + \frac{\tau_0}{\eta}\left[R - r_0 \ln R\right] + c_2 = 0 \qquad (17)$$

$$c_2 = -\frac{\Delta p}{2 \cdot \eta \cdot l}\left[\frac{R^2}{2} - r_0^2 \ln R\right] - \frac{\tau_0}{\eta}\left[R - r_0 \ln R\right] \qquad (18)$$

The speed $v_z(r)$ in sub-domain (1) for the $r \in [r_o;\ R]$ has the following expression:

$$v_z(r) = \frac{\Delta p}{2 \cdot \eta \cdot l}\left[\frac{R^2}{2} - \frac{r^2}{2} + r_0^2 \ln\frac{R}{r}\right] + \frac{\tau_0}{\eta}\left[R - r + r_0 \ln\frac{R}{r}\right] \qquad (19)$$

with $\Delta p < 0$ (pressure drop along the direction of motion).

In order to determine the radius r_o of the stopper, it is taken into account a cylinder of radius r_o placed inside the pipe, which is in equilibrium under pressure and shear forces action (fig. 2).

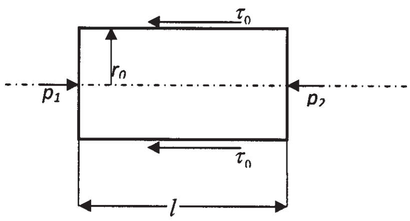

Fig. 2. Equilibrium under pressure and shear forces action for a cylinder with radius r_0 placed inside the pipe

From the dynamic equilibrium equation of pressure and friction forces on the stopper (of radius r_0 and length l), i.e.:

$$(p_1 - p_2)\pi r_0^2 = 2\tau_0 r_0 \pi l \qquad (20)$$

for the radius r_o of the fluid stopper,the following expression results:

$$r_0 = \frac{2\tau_0 l}{\Delta p},\quad \Delta p = p_1 - p_2 \qquad (21)$$

The movement speed of the fluid stopper is obtained imposing in relation (19) the condition $r = r_o$. We find the relation:

$$v_{0z}(r)=\frac{\Delta p}{2\cdot\eta\cdot l}\left[\frac{R^2}{2}-\frac{r_0^2}{2}+r_0^2\ln\frac{R}{r_0}\right]+\frac{\tau_0}{\eta}\left[R-r_0+r_0\ln\frac{R}{r_0}\right] \qquad (22)$$

Numerical application

Let us analyse the rheological fluid of the KELTAN 4200 (an etilene-co-propilene copolymer) [31,32]) that flows in a circular pipe with radius $R = 1.5\text{x}10^{-3}$ [m] and length $l = 15 \text{ x } 10^{-2}$[m]. At the temperature of $T\ H \approx 303$ [K] the dynamic coefficient of viscosity is $\eta = 77.4$ [Pas] and the gliding effort is $\tau_0 = 60$ [N/m^2].

The numerical simulation of the fluid flows takes into account the radius variation of fluid stopper r_0, of speed variation of the fluid stopper υ_{zo} and the speed variation of the fluid flows in relation to the current radius *r*. The average flow speed as a function of pressure drop Δp will be established. These numerical simulations were realized for the following pressure drops: $\Delta p_1 = 5.10^3$[N/m^2]; $\Delta p_2 = 1.10^4$[N/m^2]; $\Delta p_3 = 3.10^4$[N/m^2]; $\Delta p_4 = 5.10^4$[N/m^2]; $\Delta p_5 = 7.10^4$[N/m^2]; $\Delta p_6 = 9.10^4$[N/m^2].

The obtained results are presented in figures 3-5.

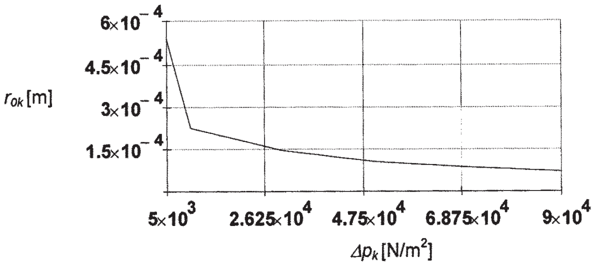

Fig. 3.Radius stopper fluid dependence on the pressure drop

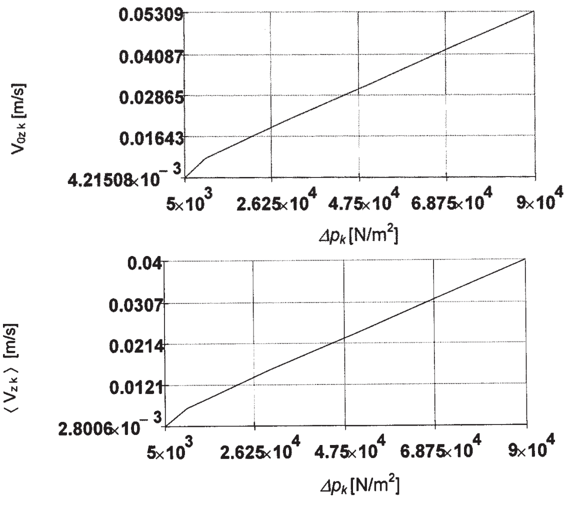

Fig. 4. Flow speed of the stopper fluid (a) and the average flow speed (b) dependence on the pressure drop

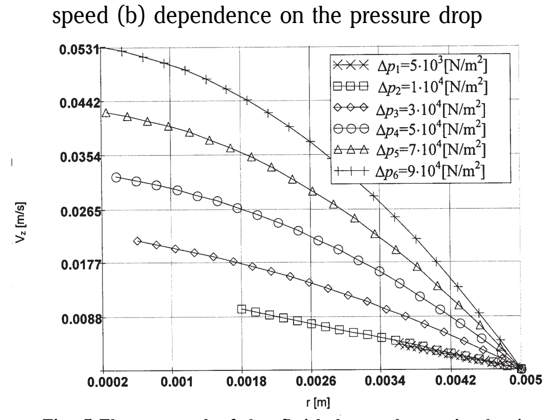

Fig. 5.Flow speed of the fluid dependence in the interval $[r_0; R]$ for various pressure drops

Conclusions

Using the Non-Standard Scale Relativity Theory approach, the specific parameters describing the flows of Bingham type rheological fluids through a circular pipe, under the action of a pressure gradient in the direction of the movement are established.

As we intended, an analytical solution is obtained. Moreover, it shows explicitly that the friction effort has two components, one of specific fluid gliding, and the other of shearing, depending on transverse speed gradient. In the central area of the fluid movements a particles agglomeration (fluid stopper) occurs, defined by a constantly moving structure.

For a polymer (rheological fluid of Keltan 4200 (an etilene-co-propilene copolymer) 3%), a numerical application is presented. The most important observations are gathered in the following sentences.

First, we find that an increase of the pressure drop implies a decrease of the fluid stopper radius and the flow speed of fluid stopper is proportional with the increase of the pressure drop. Also, the average flow speed grows with the increase of pressure drop.

Second, in the case of a constant pressure drop, the local fluid speed between the fluid stopper and walls decreases with radius, in the interval $[r_0; R]$. In practice, for a constant radius, in the stopper fluid – wall pipe border area the local speed grows with the increase of pressure drop.

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